AD-A145 554 LIFE DISTRIBUTION PROPERTIES OF DEVICES SUBJECT TO DETERIORATION RESEARCH..(U) NORTH CAROLINA UNIV AT CHARLOTTE DEPT OF MATHEMATICS M ABDEL-HAMEED 27 JUL 84 1/1

UNCLASSIFIED AFOSR-TR-84-0725 AFOSR-80-0245 F/G 12/1 NL

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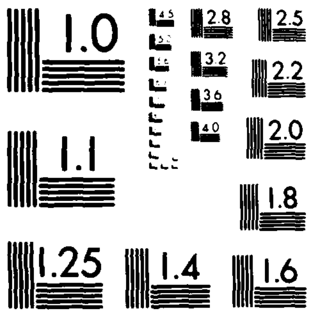
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RESEARCH PROGRESS

INTERIM SCIENTIFIC REPORT

GRANT AFOSR-80-0245

FOR THE PERIOD JULY 1, 1983 - JUNE 30, 1984

27 July 1984

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RESEARCH PROGRESS
AFOSR GRANT 80 - 0245
JULY 1, 1983 - JUNE 30, 1984

During the period July 1, 1983 to June 30, 1984, the Principal Investigator gave an invited talk on "Life Distribution Properties of Devices Subject to Deterioration" at the International Semi-Markov Processes Conference that was held at Universite´ Libre de Bruxelles under the auspices of the Bernolli Society. Moreover, he helped in organizing the session on Reliability Theory at the conference. He co-edited the proceedings of the conference he organized on Survival Models, Maintenance Policies and Life Testing. The proceedings will be published by Academic Press Publishing Company under the title "Reliability Theory and Models: Stochastic Failure Models, Optimal Maintenance Policies, Life Testing, Structure"; it is scheduled to appear on August 15, 1984. He also wrote the paper "A Power Transformation Exponential Regression Model for Cencored Failure Time Data" with Professor David Young of Brunel University, England. This paper is to be submitted for publication to Communications in Statistics; he revised the paper "Life Distribution Properties of Devices Subject to a Pure Jump Damage Process" which will appear in the December 1984 issue of Journal of Applied Probability. He is in the process of writing a paper on "Imperfect Maintenance Models" which will be submitted to Journal of Operations Research.

The Co-investigator attended numerous seminars and work group sessions on stochastic processes at University of Paris VI between March 15, 1984 and June 1, 1984. He wrote two papers:

(1) An Iterative Scheme for Approximating Optimel Replacement Policies

(2) Stability of Optimal Replacement Policies

The first paper is to appear in Reliability Theory and Models: Stochastic Failure Models, Optimal Maintenance Policies, Life Testing, Structures,

M. Abdel-Hameed, E. Çinlar and J. Quinn, Editors, Academic Press, New York, 1984. The second paper has been submitted for publication.

SUMMARY
OF RESEARCH CONDUCTES

A) LIFE DISTRIBUTION PROPERTIES OF DEVICES SUBJECT TO A PURE JUMP DAMAGE PROCESS. Suppose that a device is subject to damage, the amount of damage it suffers, over time, is assumed to be an increasing pure jump process. We denote such a process by $X \equiv (X_t, t \geq 0)$.

It is known that there exists a Poisson random measure on $R_+ \times R_+$ whose mean meausre at the point (s,z) is $ds\, dz/z^2$ and a deterministic function c defined on the positive quadrant that is increasing in the second argument such that

$$\sum_{s \leq t} f(X_{s-}, X_s) = \int_{[0,t] \times R_+} N(ds,dz)\, f(X_{s-}, X_{s-} + c(X_{s-}, z))$$

almost everywhere for each function f on $R_+ \times R_+$ with $f(x,x) = 0$ for all x in R_+. In particular, if follows that

$$X_t = X_0 + \int_{[0,t] \times R_+} N(ds,dz)\, c(X_{s-}, z).$$

The above formula has the following interpretation

$t \to X_t(w)$ jumps at s if the Poisson random measure $N(w,.)$ has an atom (s,z) and then the jump is from the left-hand limit $X_{s-}(w)$ to the right-hand limit

$$X_s = X_{s-} + c(X_{s-}, z).$$

The function $c(x,z)$ represents the damage due to a shock of magnitude z occuring at a time when the previous cumulative damage is equal to x. Assume that the device has a threshold Y and it fails once the damage exceeds or equal to Y. The failure time is therefore given by

$$\zeta = \inf\{\, t : X_t \geq Y \,\}.$$

Let $\overline{G}$ be the right tail probability of the random variable Y. Then the survival function is given by, for $t \geq 0$,

$$S(t) \equiv P(S > t)$$
$$= E[\overline{G}(X_t)]$$

We show that life distribution properties of $\overline{G}$ are inherited as corresponding properties of S. The following are samples of some of the results obtained:

(1) Theorem. Suppose that the function c above satisfies the following condition

$c(.,z):R_+ \to R_+$ is increasing for each $z \geq 0$.

Then

(i) S has increasing failure rate when $\overline{G}$ has increasing failure rate and X has a totally positive density of order two

(ii) S has increasing failure rate on the average when $\overline{G}$ has an increasing failure rate on the average and X has a totally positive density if order two

(iii) S is new better than used if $\overline{G}$ is new better than used

(2) Theorem. Suppose that the function c satisfies the following condition

$c(.,z):R_+ \to R_+$ is decreasing for each $z \geq 0$.

Then

(i) S has decreasing failure rate when $\overline{G}$ has decreasing failure rate and X has a totally positive density of order two.

(ii) X has a decreasing failure rate average when $\overline{G}$ has a decreasing failure rate average and X has a totally positive density of order two.

(iii) S is new worse than used when $\overline{G}$ is new worse than used and the function $x \to x + c(x,z)$ is an increasing function for each $z \geq 0$.

Furthermore, if the threshold depends on time and Y_t is the threshold at time $t \geq 0$. Then

$$S(t) = E\,\overline{G}(X_t, t)$$

where $\overline{G}(x,t) = P(Y_t \geq x)$ for $x,\ t \geq 0$. In this case we obtain the following result

(3) Theorem (i) Suppose that $\overline{G}(.,t)$ has increasing failure rate average for each $t \geq 0$, the mapping $\overline{G}(x,.)$ is decreasing function for each $x \geq 0$, X has a totally positive density of order two, and $c(.,z)$ is an increasing function for each $z \geq 0$. Then S has increasing failure rate average.

(ii) Suppose that $\overline{G}(.,t)$ has a decreasing failure rate average for each $t \geq 0$, the mapping $\overline{G}(x,.)$ is an increasing function for each $x \geq 0$, X has a totally positive density of order two and $c(.,z)$ is a decreasing function for each $z \geq 0$. Then S has decreasing failure rate average.

(iii) Suppose that $V = -\ln \overline{G}$ is a subadditive function on R_+^2 and $c(.,z)$ is an increasing function for each $z \geq 0$. Then S is new better than used.

(iv) Suppose that V above is a subadditive function on R_+^2 while the function $x \to x + c(x,z)$ is decreasing for each $z \geq 0$. Then S is new worse than used.

We also discuss the optimal replacement problem for such devices. Define, for $t \geq 0$,

$$Z_t = \begin{cases} X_t & , t < \zeta \\ +\infty & , t \geq \zeta . \end{cases}$$

The process $Z = (Z_t)$ is obtained by killing the process X at the failure time of the device.

A device subject to the damage process Z can be replaced before or at failure. Each replacement at failure cost c dollars, $c > 0$. The cost of a replacement before failure depends on the damage level at the time of replacement and is denoted by $c(.)$. That is to say, $c(x)$ is the cost of a replacement when the damage level at time of replacement is equal to x. Naturally, we assume that $c(x)$ is increasing and bounded above by c. Let (H_t) be the canonical history of Z. Moreover, U denotes the class of stopping times that do not exceed the life time ζ.

For any stopping time τ in U we let ξ_τ denote the expected cost of replacement per unit time. We are interested in finding the stopping time τ^* in U satisfying

$$\xi_{\tau^*} = \inf_{\tau \in U} \xi_\tau$$

That is to say, we want to find the stopping time in U that minimuzes the expected cost per unit time over U. We call such stopping time the optimal replacement time. We give conditions on the cost function $c(x)$ and the damage function $c(x,z)$ that guarantee that the optimal replacement policy is a control-limit policy.

The above results have been accepted for publication in Journal of Applied Probability and is scheduled to appear in December 1984.

B) A POWER TRANSFORMATION EXPONENTIAL REGRESSION MODEL FOR CENSORED FAILURE TIME DATA. Suppose that we have n items which are subject to failure. Let $T_1^*, T_2^*, \ldots, T_n^*$ be the random variables representing the failure times of the first, second, ..., n th item respectively. We assume that right censoring may occur because of the need for early termination of the experiment and let $T_1, T_2, \ldots, T_n$ represent the recorded survival times. Defining censoring indicator variables

$$w_i = \begin{cases} 1 & \text{if } T_i^* \text{ is uncensored} \\ 0 & \text{if } T_i^* \text{ is censored} \end{cases}$$

we have

$$T_i^* = T_i \text{ if } w_, = 1 \text{ and } T_i^* > T_i \text{ if } w_i = 0.$$

We let

$$n_u = \sum_{i=1}^{n} w_i \quad \text{and} \quad n_c = \sum_{i=1}^{n} (1-w_i)$$

denote the numbers of uncensored and censored observations respectively. Without loss of generality we label the individuals such that the first n_u items have uncensored times to failure and the remaining n_u have censored times to failure.

We now suppose that measurements are available of k explanatory variables $X_1, X_2, \ldots, X_k$. Setting $\underset{\sim}{x}' = (x_1, x_2, \ldots, x_k)$, the probability density function and survival function of T^* given $\underset{\sim}{x}$ are denoted by $f(t;\underset{\sim}{x})$ and $S(t;\underset{\sim}{x})$ respectively. If the failure rate does not depend on t, for any given $\underset{\sim}{x}$, T^* has the exponential distribution with probability density function

$$f(t;\underset{\sim}{x}) = \begin{cases} \mu_{\underset{\sim}{x}}^{-1} \exp \; (-t/\mu_{\underset{\sim}{x}}) \, , \; t \geq 0 \\ 0 \qquad\qquad \text{otherwise.} \end{cases}$$

Various models have been proposed in the literature to represent the dependence of $\mu_{\underset{\sim}{x}}$ on $\underset{\sim}{x}$. Some suthors consider the model form

$$\mu_{\underline{x}} = \lambda(1 + \underline{x}'\beta)$$

while others use the form

$$\mu_{\underline{x}} = \lambda/(1 + \underline{x}'\beta)$$

where $\underline{\beta}' = (\beta_1, \ldots, \beta_k)$ and λ is a positive constant. Both models require that the condition $\underline{x}'\underline{\beta} > -1$ must be imposed to insure that $\mu_{\underline{x}} > 0$. An alternative model which does not require a constraint to be imposed on $\underline{x}'\underline{\beta}$ is

$$\mu_{\underline{x}} = \lambda \exp \; (\underline{x}'\underline{\beta})$$

This model arises for the exponential case from the well-known family of proportional hazard repression models in which an assumed underlying hazard function is adjusted by multiplicative exponential factors to allow for the effect of the explanatory variables.

In this paper, we consider the power transformation model given by

$$\mu_{\underline{x}} = (1 + \delta\underline{x}'\underline{\beta})^{1/\delta}$$

We refer to δ as the power parameter. It is seen that when $\delta = 1$, the model corresponds to the first model discussed in the previous paragraph, second model is obtained after appropriate reparameterisation. When $\delta \to 0$ the exponential model for $\mu_{\underline{x}}$ given above is obtained.

In general, the power parameter δ as well as the coefficient vector β will have to be estimated from the data. We obtain maximum likelihood estimators for these parameters and it is shown how the estimates can be obtained using the statistical package G LIM. We also discuss the assessment of the goodness of fit of the model and numerical examples are given to illustrate the procedure.

C) STABILITY OF OPTIMAL REPLACEMENT PROBLEMS. In this work we complete and considerably extend the work reported on in preliminary fashion in section I(D) of last years proposal. Let $X = \{X_t, t \geq 0\}$ be a Hunt process with state space E. Let λ be stopping time for X and let X be the process obtained by killing X at λ. The state space of X is now $E^\Delta = E \cup \{\Delta\}$ where the augmented point Δ denotes "failure". A generalized replacement policy is a Markov time which determines a system's replacement in terms of its history. A generalized ε-optimal replacement policy at x is a generalized policy such that $E^x g(X_{\tau_\varepsilon})/E^x(\tau_\varepsilon)$ is within ε of the minimum long-run average cost for the replacement cost function g. Here E denotes expectation with respect to the process X.

A generalized optimal replacement problem (X,λ,g) (at x) is termed stable if every nearly optimal policy for the problem (X,λ,g) is nearly optimal for any "sufficiently close" problem (X^1,λ_1,g_1) and vice-versa. We show that, if life times are not permitted to vary then under suitable definitions of "closeness between problems", optimal replacement problems are stable. Examples show that, when lifetimes can vary, such is no longer the case. When X is restricted to be a one dimensional wear/damage process and the lifetimes are restricted to be threshold times, it is shown that, if the threshold is one which X misses with probability one, then every problem (X,λ,g) is stable.

These results have been submitted for publication and were part of a talk given at the 1983 Charlotte conference on Stochastic Failure Models.

D) AN ITERATIVE SCHEME FOR APPROXIMATING OPTIMAL REPLACEMENT POLICIES. Let $X = X_t$, $t \geq 0$ be a stochastic process with augmented state space E^Δ and lifetime $\lambda = \inf \{t \mid X_t = \Delta\}$. In this paper we analyze the follwoing iterative technique. Let $b_1 = E^0 g(X_\lambda)/E^0(\lambda)$. Consider the problem of maximizing the criterion $\psi(b_1,\zeta) = b_1 E^0(\tau) - E^0 g(X_\tau)$ for $\tau \leq \lambda$. If we are interested in a generalized ε-optimal policy, then tolerances $x > 0$ and $\beta > 0$ are determined in terms of X,g,λ and ε so that, if a β-optimal policy τ_1 for $\psi(b_1,\tau)$ gives $\psi(b_1,\tau) \leq \alpha$ then λ is already ε-optimal, otherwise take $b_2 = E^0 g(X_{\tau_1})/E^0(\tau_1)$ and repeat the steps on the criterion $\psi(b_2,\tau) = b_2 E^0(\tau) - E^0 g(X\tau)$. Under very reasonable assumptions on g (it must be positive and bounded away from zero), it is shown that this iterative method will supply a generalized ε-optimal replacement policy. We further implement this scheme on a computer for certain Markovian damage models. In doing so, the discrete approximations which are required are fully justified and some feeling for the speed of convergence of the procedure is obtained. The iterative scheme itself is very fast. Solving the related optimal stopping problems using dynamic programming techniques is, however, very slow.

This work constitutes a completion and considerable extension of that reported on in a preliminary fashion in section I(E) of last year's proposal. The iterative scheme itself is now seen to work for artibrary stochastic processes (as opposed to just simple wear processes). The justification of the discrete approximations required in the associated optimal stopping problems constitutes an effort in the spirit of some of the work of Whitt but applied to a problem which is outside the framework of those he has considered. The numerical implementations and simulations also represent considerable progress on the research proposed in the last paragraph of section II(E) of last year's proposal.

These results are to appear in the proceedings of the Charlotte conference on Stochastic Failure Models.

E) <u>STABILITY OF OPTIMAL STOPPING PROBLEMS</u>. For a process X and a reward function g defined on the state space of X, we have defined concepts of stability of the optimal stopping time (X,g) in previously reported work. Roughly, (X,g) is center stable if a close to optimal solution to the stopping time problem (X,g) is close to optimal for any problem (X^1,g_1) which is sufficiently "near" to the problem (X,g). The problem (X,g) is termed stable if close to optimal solutions to any sufficiently "near" problem (X^1,g_1) are close to optimal for (X,g). There are two new things to report here. First we have shown that, for $n \geq 2$, Brownian motion in R^n is <u>not</u> stable. This solves Question 2 of part II(E) of last year's proposal. We have also made headway in better understanding the metric we are using to measure distances between processes. Recall that, if $X = (\Omega, M_t, M, P^x, \theta_t, X_t)$ is a standard process in the usual sense, then $\bar{S}_X$ denotes the set of standard processes $Y = (\Omega, M_{Y,t}, M_Y, P_Y^x, \theta_t, X_t)$. For $X, Y \in S_X$ define $d(X,Y)$ by

$$d(X,Y) = \sup_{x \in E} \sup_{A \in F_1^0} |P_X^x(A) - P_Y^x(A)|.$$

We recognize d-convergence as being convergence in total variation on initial segments F_t^0. In the literature, weak convergence is what is normally looked at when one wants to consider different processes to have the same path space but different measures. However, examples show that weak convergence is not strong enough for any kind of stability for undiscounted problems with discontinuous reward functions (exactly the setting we get into when considering optimal replacement problems). What we have established about d-convergence is that, for diffusions with the same diffusion term, d-convergence is implied by uniform convergence of drift terms. However, if in the strictly elliptic case the diffusion terms are permitted to vary, then a fairly simple argument shows that the processes are singular and hence as far apart in the d-metric as is possible. This means that the study of stability within the class of diffusions is far from complete.

For compound Poisson processes, the situation is nice. Parameter convergence combined with convergence in total variation of the jump distributions is enough to get d-convergence for these processes.

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY 2b DECLASSIFICATION / DOWNGRADING SCHEDULE	3 DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S): AFOSR-TR- 84-0725
6a NAME OF PERFORMING ORGANIZATION: University of North Carolina; 6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION: Air Force Office of Scientific Research
6c ADDRESS (City, State, and ZIP Code): Dept of Mathematics & Computer Science, Charlotte NC 28223	7b ADDRESS (City, State, and ZIP Code): Directorate of Mathematical & Information Sciences, AFOSR, Bolling AFB DC 20332
8a NAME OF FUNDING/SPONSORING ORGANIZATION: AFOSR; 8b OFFICE SYMBOL (If applicable): NM	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: AFOSR-80-0245
8c ADDRESS (City, State, and ZIP Code): Bolling AFB DC 20332	10 SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO 61102F; PROJECT NO 2304; TASK NO A5; WORK UNIT ACCESSION NO.

11 TITLE (Include Security Classification)
RESEARCH PROGRESS, INTERIM SCIENTIFIC REPORT, GRANT AFOSR-80-0245, FOR THE PERIOD JULY 1, 1983 - JUNE 30, 1984.

12 PERSONAL AUTHOR(S)
Mohamed Abdel-Hameed

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Interim	FROM 1/7/83 TO 30/6/84	27 JUL 84	11

16 SUPPLEMENTARY NOTATION

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

Research during this period concentrated on the following areas: (1) life distribution properties of devices subject to a pure jump damage process; (2) a power transformation exponential regression model for censored failure time data; (3) stability of optimal replacement problems; (4) an iterative scheme for approximating optimal replacement policies; and (5) stability of optimal stopping problems. This report summarizes progress in these areas.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL: CPT Brian W. Woodruff	22b TELEPHONE (Include Area Code): (202) 767- 5027; 22c OFFICE SYMBOL: NM

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

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